Volatility Noise*

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Abstract

This study shows that fitting errors of equity option implied volatility surfaces are informative about limits of arbitrage. For each stock and day, we quantify the goodness of fit between the observed implied volatilities of all available options and the corresponding estimates from OptionMetrics' smoothed volatility surface using root-mean-square errors. In the cross-section of stocks, this error metric increases in idiosyncratic stock volatility and several measures of option and stock illiquidity. Based on these insights, we propose a measure for market-wide limits of arbitrage given by the value-weighted average of the stock-specific fitting errors. This measure of *volatility noise* peaks during episodes of market distress and exhibits sensible correlations to standard economic state variables like the market return, the TED spread, or the VIX. It co-moves as well with both the conceptually similar *(treasury) noise measure* proposed by Hu, Pan and Wang (2013) and a mispricing measure based on covered interest rate parity deviations in FX markets, but volatility noise still contains unique information. Confirming this view, we find that among these measures only volatility noise constitutes a priced risk factor in monthly returns of managed equity portfolios.

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1 Introduction

Arbitrage activities are a fundamental cornerstone of efficient financial markets. In a world without frictions, arbitrageurs trade against arbitrarily small mispricings for the prospect of a risk-free profit. These trades drive prices closer to their fundamentals, thereby increasing market efficiency. But in practice, there exist numerous potential limits of arbitrage that restrict informed agents from trading against misaligned prices. For example, it might not be possible to construct a perfectly risk-free arbitrage portfolio, maybe due to the non-availability of securities with the needed payout profile or the inability to adjust portfolio compositions in continuous time. In addition, trading may be costly due to transaction costs or high capital requirements. The literature on intermediary asset pricing[1] highlights the important role of the capital resources available to intermediaries like banks, dealers, and fund managers. These market participants have the necessary expertise to run arbitrage strategies, and this ability may be limited if they face capital constraints. Further, Shleifer and Vishny (1997) argue that arbitrageurs are highly specialized, holding undiversified portfolios. Thus, both systematic and idiosyncratic volatility matter to these arbitrageurs and may lead to a decline in arbitrage activities. All these mechanisms result in a tight connection between the magnitudes of mispricing and financial frictions.

In this study, we analyze the link between misaligned equity option prices and overall limits of arbitrage. The option market is particularly suitable for this kind of study, as there are many different options per stock available, sharing the same source of fundamental risk. Even more important, equity options are written on a broad spectrum of different stocks. So, we can exploit that in times of high frictions, stock-specific measures for option mispricing tend

[1] See for example He and Krishnamurthy (2013); Chen, Joslin and Ni (2016); He, Kelly and Manela (2017).

to peak simultaneously, for example due to an overall shortage of arbitrage capital that affects the stock market as a whole. We therefore propose *volatility noise* as the value-weighted average over the stock-specific fitting errors. This aggregation is a crucial aspect of our approach, as we are able to eliminate idiosyncratic components, such that the resulting measure is particularly informative about market-wide limits of arbitrage.

We quantify the stock-specific option mispricing on each trading day by comparing the observed implied volatilities with benchmark volatilities derived from a smoothed volatility surface. In the cross-section of stocks, these pricing errors increase in idiosyncratic volatility and illiquidity of both the stock and the associated options. This result gives a first indication about the close link between option price mismatches and limits of arbitrage. Consequently, a time-series analysis shows that the resulting aggregate volatility noise measure is able to identify periods of market distress. It peaks during NBER recessions, financial crises, and salient macro-economic events. We find sensible correlations between volatility noise and important economic state variables like the TED spread, the stock market return, the VIX, and market liquidity, with particularly strong links to state variables of the equity market. In addition, there is a close connection to the *(treasury) noise measure* proposed by Hu, Pan and Wang (2013), which is defined as the root-mean-square error between yields of U.S. Treasury bonds and a smooth yield curve. We also find high correlations to a measure for arbitrage opportunities on the foreign exchange market, as introduced by Pasquariello (2014) and Klingler (2016), which quantifies deviations from the covered interest rate parity (CIP). All three measures are conceptually similar, but based on completely different markets, so their high correlation indicates that the measures indeed all account for limits of arbitrage. Finally, in a vector autoregressive analysis, we find that volatility noise Granger-causes treasury noise without reverse causality, whereas we find mutual Granger-causality between volatility noise and the CIP measure.

These findings suggest that volatility noise contains unique information about limits of arbitrage, especially for equity markets. As mispricing due to constrained arbitrageurs poses an important source of risk, volatility noise as a measure of limits of arbitrage should carry a risk premium in expected returns. We analyze monthly returns of equity mutual funds from the Morningstar database to investigate this hypothesis. Fund managers may willingly choose to take higher risks to beat their benchmark, and active management by itself may result in an additional exposure to liquidity risk, which is of course an important dimension of the limits of arbitrage (Dong, Feng and Sadka, 2013). Therefore, we expect to find pronounced pricing effects of arbitrage risk in mutual funds. And indeed, running a monthly-rebalanced portfolio sort on volatility noise betas, we find a significant long-short return of about 3.6% per year. In addition, we find significant alphas for the Fama and French (1993) three-factor, Carhart (1997) four-factor, and Fama and French (2015) five-factor model, so the premium on volatility noise cannot be explained by these standard risk factors. The corresponding analyses for treasury noise and the CIP measure yield no significant premium, which confirms the intuition that volatility noise represents an additional source of risk in excess of the other measures of limits of arbitrage.

Our paper contributes to the extensive literature on limits of arbitrage, which argues that arbitrage activities may be prevented if markets are, for example, illiquid or volatile (see, for example, Shleifer and Vishny, 1997; Kyle and Xiong, 2001; Gromb and Vayanos, 2002; Pontiff, 2006; Mitchell, Pedersen and Pulvino, 2007; Cao and Han, 2016). Consequently, market efficiency declines and prices move away from fundamentals.

We are not the first to infer the extent of financial frictions from such price discrepancies. Hu, Pan and Wang (2013) construct their noise measure from deviations between U.S. treasury yields and the respective yield curves. Similar ideas have been applied to differences between theoretical and actual forward exchange rates (Pasquariello, 2014; Klingler, 2016), index and equity variance risk premia (Barras and Malkhozov, 2016), and pricing discrepancies of ETFs and their underlying components (Chacko, Das and Fan, 2016), for example. Concerning option markets, several studies analyze violations of the put-call parity and find connections to short-sale constraints (Ofek, Richardson and Whitelaw, 2004) and liquidity (Kamara and Miller, 1995). Golez, Jackwerth and Slavutskaya (2017) propose a funding liquidity measure that is also based on put-call parity relations. These studies rely on perfectly matching put-call pairs and usually depend on restrictive assumptions on transaction costs or market maker positions. In contrast, we construct our measure of volatility noise from all given options without further assumptions, thus taking advantage of the full information available.

We contribute naturally to the literature on asset pricing implications of implied volatility characteristics. Innovations in the level of volatility surfaces predict returns (Dennis, Mayhew and Stivers, 2006; Banerjee, Doran and Peterson, 2007; An et al., 2014), and so does the smile (Xing, Zhang and Zhao, 2010; Yan, 2011) and term-structure of implied volatility (Vasquez, 2015). There is also a growing strand of literature on the predictability of volatility spreads (see, for example, Bali and Hovakimian, 2009; Cremers and Weinbaum, 2010; Doran, Fodor and Jiang, 2013). Fournier and Jacobs (2017) relate the variance risk premium to financial constraints, Duan and Wei (2009) find a relation between the systematic risk of a stock and the level and slope of its associated implied volatility surface. Recently, Schlag, Thimme and Weber (2017) introduce the concept of an implied volatility duration and establish a connection to the early resolution premium. So there are many studies on asset pricing implications of implied volatilities, but we are, to the best of our knowledge, the first to analyze the fitting errors of volatility surfaces, establishing a link to limits of arbitrage.

Finally, as we find a risk premium on volatility noise in the cross-section of fund returns, we contribute as well to the literature on liquidity risk in managed portfolios (Billio, Getmansky and Pelizzon, 2010; Sadka, 2010; Cao et al., 2013; Dong, Feng and Sadka, 2013). While this literature focuses predominantly on market liquidity, volatility noise measures general limits of arbitrage, including the effects of funding liquidity and idiosyncratic volatility, for example.

The rest of this paper is structured as follows. In Section 2, we describe the option data and analyze the goodness of fit between implied volatilities and the corresponding smoothed volatility surface. Section 3 covers the aggregation of individual fitting errors to market-wide volatility noise and the time-series properties of this measure. Further, we analyze pricing implications of volatility noise risk for mutual fund returns. Section 4 highlights the importance of the chosen aggregation procedure and confirms the robustness of our results, Section 5 concludes the paper.

2 Fitting Errors of Implied Volatilities

2.1 Option data

Our main data source is the OptionMetrics Ivy database, which covers the entire U.S. listed equity and index option market. The sample period begins in January 1996 and ends in April 2016. In addition to daily option trading information and closing bid and ask prices, this database contains implied volatilities derived from a binomial tree model (following Cox, Ross and Rubinstein, 1979). Even more, OptionMetrics provides smoothed volatility surfaces for each underlying stock and day, as long as there are enough historical option prices for a reliable estimation. For a hypothetical option j, the corresponding point $\bar{\sigma}_j$ on the volatility surface is defined as a weighted sum over the actually observed volatilities $\sigma_1, \ldots, \sigma_N$ from all options written on the underlying stock:

$$\bar{\sigma}_j = \frac{\sum_i \Phi(i,j)\, \nu_i\, \sigma_i}{\sum_i \Phi(i,j)\, \nu_i}. \tag{1}$$

The weights depend on a distance function Φ, which takes larger values when options i and j are more similar with respect to time to maturity, moneyness, and option type (i.e., call or put). Importantly, the weight of an implied volatility σ_i is also proportional to the corresponding option vega ν_i, which quantifies the sensitivity of option prices to volatility. If vega is small, large changes in implied volatilities result in small price changes. By implication, small measurement errors in prices translate to large errors in implied volatilities.[2] To counteract this effect, such implied volatilities receive a smaller weight. For more technical details on the derivation of the volatility surface, see Appendix B.

In our main study, we focus on standard equity options, but we also analyze volatility fitting errors of index options as a robustness analysis in Section 4. We exclude options with less than 30 days and more than a year to maturity and restrict the absolute value of options' deltas between 0.2 and 0.8, as the given volatility surface does only cover this region.[3]

[Table 1 about here.]

Summary statistics on our sample are shown in Table 1. On average, the sample contains 3 260 stocks per year with 24 options per stock-day. This amounts to about 16 million different option series per year. Overall, our sample contains more than 340 million daily

[2] See Hentschel (2003) for a formal analysis on the impact of measurement errors on the estimation of implied volatilities.

[3] The surface actually covers options with up to 730 days to maturity, but given the illiquidity of these long-term options, we only consider times to maturity up to a year.

option observations.

2.2 The goodness of fit of the volatility surface

To compare observed implied volatilities with their counterparts from the smoothed volatility surface, we need a fitted volatility as a reference point for each available option. OptionMetrics' volatility surface is given on 130 equally-spaced grid points spanning expirations between 30 and 730 calendar days, and deltas between 0.20 and 0.80 (negative deltas for puts), separately for calls and puts, on a daily basis for each underlying. We define the reference volatility of a given option as the estimate from a linear interpolation between the four nearest grid points in the moneyness-expiration dimensions.[4] In principle, we could calculate the exact smoothed values using Eq. (1) for each observed option instead, but this would be computationally more intensive. Since the given grid points of the volatility surface are narrow, the interpolation has only a negligible impact on the resulting error metrics.

To get an intuition about the interpolation procedure, we visualize the calculation for an exemplary stock-day in the following.[5] Fig. 1 shows all observed implied volatilities considered in this example.

[Figure 1 about here.]

The top panel shows call options, the bottom one put options. As result of the fixed issuing schedule of the standard listed options, the observed volatilities lie on a rather uniform grid and we only have four different times to maturity.

[4] OptionMetrics uses a similar approach to calculate implied volatilities for the pricing of standardized options.

[5] The shown implied volatilities belong to options written on *Teladoc, Inc.* as of January 29, 2016.

[Figure 2 about here.]

Fig. 2 visualizes the calculation of the volatility fitting errors for the subsample of put options with 77 days to maturity (as indicated by blue dots in Fig. 1). The dotted line shows the true, smoothed volatility surface, which we have calculated on our own using Eq. (1). The solid line results from the linear interpolation between the grid points provided by OptionMetrics, which in turn are drawn as black squares. In our actual calculation, we use a two-dimensional interpolation, while Fig. 2 only shows an interpolation along the moneyness dimension for illustrative purposes. In any case, the deviation between the true volatility surface and the linearly interpolated surface is indeed very small, which we see as justification for the chosen approximation. Finally, the option-specific fitting error is simply given by the distance between the observed volatilities and the linearized surface, as illustrated by the red bars.

We report average volatility fitting error per moneyness-maturity category in Panel A of Table 2. The fitting errors are highest for short maturities, and in particular for out-of-the-money call and in-the-money put options. On the other hand, the fit is best for at-the-money options. The patterns in average open interest and trading volume are quite different, as shown in Panel B and Panel C, respectively. Open interest is both concentrated at short and long maturities, which applies also in some extent to trading volume. In addition, both variables are decreasing in moneyness for both call and put options. So there seems to be no direct link between trading activity and volatility fitting errors. On the other hand, we do observe such a connection to the number of options, as shown in Panel D. We observe more options for short maturities, relatively few of them are at-the-money. This highlights a feature of the smoothed volatility surface: If there are only few options with similar strike and maturity, the surface is dominated by these options and so the fit is better for the corresponding implied volatilities.

[Table 2 about here.]

In addition to these results on time-series averages, Fig. 3 shows the shape of fitting errors on several exemplary dates. On each of these dates, we group all available options by their call-equivalent delta (delta for calls, one plus delta for puts) and calculate quantiles of the fitting errors across all underlying stocks. The top left panel shows the pattern on January 3, 2006, an arbitrarily chosen day during normal times. The corresponding fitting errors are quite low and there is no distinctive pattern in the moneyness dimension. In contrast, the top right panel corresponds to a day during the financial crisis, where the fitting errors are very large in comparison to the ones in normal times. It appears that the implied volatilities of out-of-the-money call options (and in-the-money put options) are particularly poorly fitted, confirming the results given in Table 2. Finally, the bottom panels show that the distribution can rapidly change from one day to another: the bottom right panel corresponds to the 2010 flash crash, whereas the one on the bottom left shows data from the preceding day. Apparently, the large price movements during the crash resulted in higher uncertainty and mispricing in option markets.

[Figure 3 about here.]

These examples are already indicative for a close link between fitting errors and limits of arbitrage, or economic conditions in general. We investigate this connection more closely on an aggregate level in Section 3. But first, we analyze the cross-sectional relation between fitting errors on the stock-level and stock characteristics.

2.3 Volatility fitting errors on the stock-level

We quantify the overall fitting error for stock i on day t with the root-mean-square error based on all available options written on this stock:

$$error_{i,t} = \sqrt{\frac{1}{n_{i,t}} \sum_{j=1}^{n_{i,t}} \left(\sigma_{j,t} - \hat{\sigma}_{j,t}\right)^2}, \tag{2}$$

where $\sigma_{j,t}$ is the actually observed implied volatility of option j, $\hat{\sigma}_{j,t}$ is the corresponding fitted value from the interpolated volatility surface, and $n_{i,t}$ is the number of options written on stock i on day t.

This metric summarizes how well the implied volatilities of all options written on a given stock can be described by the smoothed volatility surface. We hypothesize that the goodness of this fit strongly depends on limits of arbitrage, as large deviations between implied volatilities and the volatility surface is indicative for "very good deals", if not downright arbitrage opportunities. In the cross-section, for some stocks there might be more impediments to arbitrage activities, for example due to illiquidity or higher idiosyncratic volatility (cf. Shleifer and Vishny, 1997). To shed light on this relation, we form monthly decile stock portfolios by the average daily stock-level fitting error over the preceding month. As shown in Table 3, stocks with higher fitting errors belong to smaller firms and are less liquid: Trading volume is decreasing, relative bid-ask spreads and Amihud (2002) illiquidity are increasing in the fitting error portfolio rank. The same is true for the idiosyncratic volatility, which we calculate as the standard deviation of the residuals from the Fama and French (1993) factor model estimated on daily stock returns over the previous month. This definition of idiosyncratic volatility follows Ang et al. (2006). As discussed by Shleifer and Vishny (1997), both illiquidity and idiosyncratic volatility are proxies for limits of arbitrage, so these results are in line with our hypotheses.

[Table 3 about here.]

The bottom part of Table 3 shows average option characteristics written on the stocks. Like for the stock characteristics, we find a positive connection between fitting errors and option illiquidity, as indicated by decreasing open interest and trading volume, and increasing option bid-ask spread relative to the option price.

3 Volatility Noise and Limits of Arbitrage

3.1 Aggregated fitting errors

The stock-level volatility fitting errors, as given by Eq. (2), tell us how well the actually observed implied volatilities can be described by the smoothed volatility surface. As part of the deviations from the fitted values may represent arbitrage opportunities, an aggregate measure of fitting errors may give an indication about market-wide limits of arbitrage.

We construct our measure of *volatility noise* simply as the average over all stock-individual fitting errors, weighted by the value of open interest. Using value weighting, we focus on stocks with more liquid options, which potentially puts less emphasis on measurement errors and unsystematic fitting errors. For the same reason, we exclude the daily 5% lowest and highest stock-level fitting errors from averaging. The chosen weighting scheme and filter criteria yield a less volatile measure, but our results do not depend on these choices.[6]

[6] See Section 4 for a robustness analysis on these alternative specifications.

[Figure 4 about here.]

Fig. 4 shows the resulting time series of volatility noise along with several important macroeconomic events. Strikingly, our measure peaks during the NBER recession periods and large-scale episodes like the European debt crisis, and matches smaller events of market distress as well. From 2010 onwards, the measure exhibits a strong upward trend, which we attribute to increasing arbitrage impediments due to progressing regulatory constraints. One example of such regulatory changes is the Dodd-Frank act, which became effective on July 21, 2010.[7] In addition, in 2011 the Fed announced the implementation of more stringent rules on capital requirements following Basel III (cf. Wyatt, 2011). Furthermore, according to Banerji (2017), at least 6 major option market makers stopped their trading activity since 2012, which could also explain part of the increasing trend in option mispricing. In line with Fig. 4, Table 4 shows that the average level of volatility noise of 1.181% between 2012 and 2016 is more than twice as high as the level in the periods before the financial crisis in 2008. Between 2004 and 2007, volatility noise reaches its lowest average level of 0.347%. A similar pattern can be observed for the standard deviation: Volatility noise is more volatile during periods of market distress. Finally, the last column shows that volatility noise is quite persistent, with an overall first-order autocorrelation coefficients of 0.913.

[Table 4 about here.]

The descriptive analysis of the time-series of volatility noise already points to a potential connection to the overall market state. To investigate this link, we present correlations of monthly changes of volatility noise and several well-known economic state variables in Table 5,

[7] In the same spirit, Cumming, Dai and Johan (2017) show that the Dodd-Frank act has a strong impact on hedge fund performance, risk, and fund flows.

Panel A. The first variable is the noise measure of Hu, Pan and Wang (2013), which we call *treasury noise* in the following to avoid misconceptions.[8] Treasury noise is constructed as root-mean-square error of treasury yield curves, and so it is both by construction and by intuition quite similar to volatility noise. A closely related measure for limits of arbitrage can be constructed in the foreign exchange market as well, exploiting the well-known covered interest rate parity. More precisely, Pasquariello (2014) and Klingler (2016) propose a measure for CIP deviations, which is given by the absolute log-difference between CIP-implied theoretical and actually observed forward exchange rates for several currencies on different time horizons.[9] As it reflects mispricings in the foreign exchange market, just like volatility and treasury noise quantify price deviations in the options and treasury market, the CIP measure is a natural additional reference point for our analysis. For consistency, we call it *FX noise* in the following, although it differs in some details of construction from the other two noise measures. In any case, given the conceptual similarities between these measures, we expect to find a close connection between them, and indeed, we find high correlations between 21 and 33 percent between monthly changes in these variables.

Volatility noise is positively related to market and funding liquidity, as measured by average relative bid-ask spreads, Amihud (2002) illiquidity, and the TED spread. It is also sensitive to increases in expected volatility and market downturns, as indicated by the large correlations of 0.40 to changes in the VIX and −0.29 to the market return. Correlations to interest rates and the on-the-run premium have intuitive signs, but are only small in magnitude. Remarkably, the correlation patterns between all three noise measures and the remaining state variables are very similar, both in terms of sign and magnitude. This indicates that

[8] The time-series of treasury noise between 1987 and 2014 may be obtained from Jun Pan's homepage, `http://www.mit.edu/~junpan/`. We thank Stefan Fiesel for providing an augmented time-series that covers the time span until December 2016. See Fiesel, Uhrig-Homburg and Brunzel (2017) for further details.

[9] We thank Sven Klingler for providing us the corresponding time series of the CIP measure.

all three noise measures quantify similar frictions and one may ask whether and how these measures differ.

[Table 5 about here.]

As a first answer to this question, we also analyze correlations for *weekly* changes in Table 5, Panel B. The correlations between volatility noise and both the market return and changes in the VIX are still quite high, especially in comparison to the corresponding correlations between these variables and the other noise measures. On the other hand, the connection between volatility noise and the state variables coming from the fixed-income market are much weaker. The most striking difference to monthly correlations is the very low correlation between volatility noise and treasury noise of only 2 percent, whereas the link between volatility noise and FX noise remains unchanged. These results lead us to expect that volatility noise contains unique information beyond treasury noise, particularly for equity markets. On the other hand, there is an undisputedly strong connection between volatility noise and FX noise.

To shed more light on the intertemporal associations between the three noise measures, we consider vector autoregression (VAR) models on both a monthly and weekly frequency. First, we apply a logarithmic transformation on all three time series to reduce the impact of the large peaks during the crisis periods and to mitigate heteroskedasticity. In addition, the VAR model in logarithms results in a much better fit in terms of root-mean-square errors and coefficients of determination. The optimal lag length for the monthly and weekly frequency according to Akaike's information criterion (AIC) is 4 months and 15 weeks, respectively.[10] Table 6 shows

[10] The Bayesian information criterion (BIC) suggests less lags, but the corresponding VAR models are not well-specified, as there is significant serial correlation in the regression residuals.

the results from pairwise Granger causality tests between the three noise measures.[11] On both frequencies, we find that only volatility noise significantly Granger-causes the other two variables. Whereas FX noise also Granger-causes volatility noise, we find no causal impact on treasury noise, which in turn does not Granger-cause any of the other noise measures. In combination with the previous results on correlations, we conclude that both volatility and FX noise capture new, complementary information beyond treasury noise.

[Table 6 about here.]

3.2 Pricing of volatility noise risk in equity mutual fund returns

As shown in the previous analyses, volatility noise exhibits a close link to the economic state, peaking in periods of limited arbitrage capital and illiquidity, with a particularly close relation to equity-related state variables. For this reason, we expect that volatility noise carries a risk premium in the cross-section of equity mutual funds.

Managed portfolios are a particularly well suited asset class for this research question, as fund managers are willing to accept higher risks to maximize their salary (cf. Massa and Patgiri, 2009). Further, Dong, Feng and Sadka (2013) argue that informed fund managers' ability to outperform strongly depends on market liquidity, which results in additional liquidity risk exposure. Consequently, fund returns may exhibit a much stronger link to frictions, and consequently volatility noise risk, than the underlying stock returns.

In the following, we focus on mutual funds, as these funds have much stricter reporting and transparency rules than hedge funds. For example, while hedge funds may choose to use

[11] All considered time series are stationary, with the sole exception of the monthly time series of log-treasury noise, which is integrated of order one. We rely on the procedure proposed by Toda and Yamamoto (1995), which enables us to test for Granger-causality between integrated processes.

redemption gates during liquidity crises (cf. Teo, 2011), mutual funds have no such possibility to prevent costly capital withdraws. Consequently, liquidity risk exposures have a much more pronounced and immediate impact on mutual fund returns by comparison.

Our fund sample comes from the Morningstar mutual fund database. We select all funds with an investment style focused on US equities, as indicated by the Morningstar category, excluding funds of funds and index funds. In line with Pástor, Stambaugh and Taylor (2015), we remove funds younger than 3 years or less than \$15 million assets under management to address the incubation bias in the early phase of a fund's lifespan. For more details on the sample construction, see Appendix C. The final sample covers the the period between February 1996 and April 2016, with 1586 funds per month on average.

As the time series of volatility noise is highly autocorrelated, we extract volatility noise shocks with a time-series model. Specifically, we fit a second-order autoregressive model without intercept to monthly changes in volatility noise:

$$\Delta vn_t = \theta_1 \cdot \Delta vn_{t-1} + \theta_2 \cdot \Delta vn_{t-2} + \varepsilon_t \tag{3}$$

The volatility noise shock is then defined as the negative residual from this regression: $\nu_t \equiv -\varepsilon_t$. We multiply the residuals with negative one to simplify the interpretation: Just like in the case of a standard risk factor, a positive shock is a desirable outcome. For comparison, we analyze shocks to treasury and FX noise as well, which are simply defined as the negative first difference of the respective time series. All of the chosen models are optimal according to both AIC and BIC among all ARIMA model with or without intercept.

We form monthly equally-weighted decile portfolios on ex-ante noise betas, which we estimate from rolling regressions using a window width of 36 months:

$$r_t^i = \alpha + \beta_i^N \nu_t + \beta_i^{MKT}\ MKT_t + \beta_i^{SMB}\ SMB_t + \beta_i^{HML}\ HML_t + \varepsilon_{i,t}, \tag{4}$$

where r_t^i is excess returns of fund i and ν_t is the shock in the respective noise measure. In addition, we control for the Fama-French factors MKT_t, SMB_t, and HML_t.[12]

[Table 7 about here.]

Table 7 shows the results from the portfolio sort on volatility noise betas. First of all, the ex-ante betas estimated from the rolling window regression are quite symmetric around zero, covering a range from −3.72 to 4.06. The ex-post noise betas, which are estimated from regressions of the full time-series of portfolio returns, increase quite monotonously in the portfolio rank and are throughout positive. So while ex-ante betas appear to be a good estimator for ex-post betas when it comes to the relative size, their levels differ. The fact that all portfolio noise betas are positive indicates that volatility noise shocks indeed constitute an important source of risk that cannot be easily avoided. Portfolio excess returns and alphas from standard factor models are predominantly increasing in portfolio rank, with a statistically significant long-short return of 0.30 percent per month, which corresponds to an also economically significant premium of 3.60% per year. In addition, we find significant alphas between 0.25 and 0.28 percent.

Table 7 also shows further characteristics of the decile portfolios. While all portfolios have a market beta close to one, betas with respect to the HML and SMB factor exhibit a distinct hump and u-shape, respectively. This finding highlights the importance of the chosen risk

[12]Fama-French research factors are obtained from Kenneth French's data library, `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

factors as control variables, as the effect of volatility noise may otherwise be hidden due to the unobserved variation in the corresponding factor exposures. Finally, as shown in the last part of Table 7, neither of the funds' age, assets under management, expense ratio, and turnover ratio does vary systematically between the portfolios, which shows that the found premia are not driven by differences in fund characteristics.

[Table 8 about here.]

To conclude this analysis, we run Fama-MacBeth regression of individual fund returns on ex-post portfolio betas, following Fama and French (1992). That is, in a given month, we assign each fund the ex-post beta of the portfolio to which the fund is allocated to in that month. Then we regress next month's fund excess returns on these portfolio betas. We report time-series average of the resulting regression coefficients in Table 8. The first column correspond to the already discussed portfolio sort on volatility noise betas. In the first specification, we regress fund returns on the betas corresponding to volatility noise shocks and the Fama-French factors. Here we find a significant coefficient of 0.19 for volatility noise. Coefficients for the other factors are all positive, as expected, but none of them is statistically significant. The second specification shows that these findings are robust to the inclusion of the funds' size and age, the expense ratio and the turnover ratio as control variables.

For comparison, we carry out the same analyses for treasury and FX noise, as well. In the corresponding portfolio sorts, we find no significant long-short returns or alphas (see Table A1 and Table A2 in the appendix). Columns (3) and (4) of Table 8 show the results from Fama-MacBeth regressions corresponding to treasury noise, results for FX noise are shown in columns (5) and (6). In line with the portfolio sorts, we do not find any significant premia on these noise measures.

These analyses show that volatility noise captures indeed unique information beyond the shocks in treasury and FX noise, as already conjectured in the previous section. As volatility noise is constructed from equity options, and given the significant premium in equity mutual fund returns, we conclude that volatility noise is, in particular, tightly linked to frictions in the equity market, for which investors demand a substantial premium.

4 The Importance and Robustness of Aggregation

We define volatility noise as the value-weighted average of stock-specific fitting errors. As an alternative specification, one could also switch the order of aggregation, that is, analyze the fitting errors of options written on a broad value-weighted market index. Following this idea, we define *index noise* as the root-mean-square error based on all options written on the S&P 500 index.[13] In the first line of Table 9, we show summary statistics on this measure.

[Table 9 about here.]

The mean level of index noise is 0.29%, which is less than a half of the average level of volatility noise from stock options, which is shown in the second line. The standard deviation is 0.13%, which is about a third of the one of stock noise. These findings are not particularly surprising as index options are much more liquid than the average stock option. The last two columns show that the correlations with treasury noise and volatility noise are very low. As shown in Table 10, correlations between index noise and the considered economic state variables are low as well. Many correlations, like with the TED spread or the two liquidity measures, have even an unreasonable sign.

[13] As in our baseline specification, we only consider options with an absolute value of delta between 0.2 and 0.8, and maturities from 30 days to one year.

[Table 10 about here.]

Finally, we form fund portfolios on index noise, using the same approach as for the baseline volatility noise (see Section 3.2). The first column of Table 11 shows average returns and alphas of the resulting long-short portfolio. None of them is statistically different from zero, the three-factor alpha is even weakly negative. On the one hand, these analyses show that there are considerable fitting errors for individual underlying assets, even if the associated options are outstandingly liquid, as it is the case for the considered index options. On the other hand, although returns of the S&P 500 are basically by definition linked to systematic risk, the corresponding fitting errors appear to be predominantly idiosyncratic, as they neither exhibit high correlations with economic state variables nor carry a risk premium in expected returns. This result highlights the suitability of the cross-section of options to derive a measure of systematic limits to arbitrage. Given the multitude of options, we are able to quantify fitting errors at the stock level, which may depend on idiosyncratic influences, as we have seen even for the S&P 500, but share a common exposure to market-wide frictions. Consequently, by forming an average over the stock-specific fitting errors, we effectively cancel the idiosyncratic effects and get a meaningful measure of the systematic limits of arbitrage.

[Table 11 about here.]

Thus, the aggregation of individual fitting errors is a crucial component of our measure, but the specific way of aggregation is not important. In our baseline specification, we remove the 5% lowest and highest fitting errors, and use value weighting to aggregate stock-individual errors to market-wide volatility noise. We chose this specification to minimize the influence of outliers and reporting errors, but the general properties of our measure do not depend on these details. In the first row of Fig. 5, we show time-series plots of index noise and the baseline stock volatility noise, the second row shows stock volatility noise measures resulting from an aggregation with equal weighting and without trimming of extreme values, respectively. Consistent with our previous analysis, the time-series of index noise stands out against the aggregated stock volatility noise measures, which in turn are all quite similar. The corresponding descriptive statistics in Table 9 and long-short portfolio characteristics in Table 11 lead to the conclusion that the trimming of extreme values significantly improves the informational content of volatility noise, whereas the chosen weighting scheme has a weaker effect. Overall, all three specifications exhibit a clear link to limits of arbitrage and are consistently priced in the cross-section of equity mutual fund returns.

[Figure 5 about here.]

So far, all volatility noise measures are derived from all available options. Then again, the literature[14] documents differences in the demand for call and put options, resulting in option-type specific pricing effects. Mapping this finding on our setting, volatility noise solely derived from call and put options, respectively, may measure distinct aspects of limits of arbitrage.

Based on the associated risk premia, as shown in Table 11, these volatility noise measures appear to be quite similar. For both measures we find significant five-factor alphas of similar size, the long-short portfolios corresponding to call volatility noise also earns a weakly significant average excess return. However, the corresponding time-series plots in Fig. 5 and the descriptive statistics in Table 9 uncover several differences between these measures. The mean, volatility, and quantiles of put volatility noise are all higher than for our baseline specification, which in turn lie above the values for call volatility noise. Also, the correlations

[14] See for example Bondarenko (2014), among others.

to treasury noise and to the baseline volatility noise are clearly higher for the put-based measure. A more detailed analysis whether and to what extent fitting errors in call and put options cover different aspects of limits of arbitrage is an interesting question for future research.

5 Conclusion

In this paper, we analyze deviations between equity option implied volatilities and the corresponding fitted values from a smoothed volatility surface. Such deviations may be the result from mispricing and therefore, they should contain information on the limits of arbitrage. Using root-mean-square errors for aggregation, we find that fitting errors on the stock-level are associated with smaller firm size, lower liquidity and higher idiosyncratic risk, underpinning the link to limits of arbitrage. Consequently, we propose a market-wide measure for limits of arbitrage by means of a value-weighed average over the stock-individual mispricings. This measure of volatility noise co-moves with the business cycle, peaks during phases of market distress, and is tightly connected to economic state variables.

Finally, asset pricing tests on the time-series of volatility noise show that it constitutes a priced risk factor in monthly returns of equity mutual funds. In particular, these analyses imply that volatility noise is a unique source of risk beyond treasure noise, FX noise, and standard equity risk factors.

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A Further Analyses

Table A1: Mutual fund portfolio sort on treasury noise betas

This table shows excess returns, alphas, and characteristics of mutual fund portfolios formed on treasury noise betas. Returns and alphas are given in monthly percent, Newey and West (1987) t-statistics are shown in parentheses. Alphas are intercepts of regression of monthly returns on risk factors. Three-factor alphas are based on the Fama and French (1993) risk factors, four-factor alphas include the additional momentum factor of Carhart (1997). The risk factors of Fama and French (2015) give rise to the five-factor alpha. Ex-ante betas are calculated as the mean of the rolling betas used to form the portfolios, ex-post betas are determined in a regression over the full time-series of portfolio returns. The last part of the table shows the average age, assets under management, expense ratio, and turnover ratio of the funds in each portfolio.

Portfolio	1	2	3	4	5	6	7	8	9	10	10–1
Excess return	0.41	0.46	0.41	0.42	0.43	0.44	0.45	0.50	0.52	0.63	0.22
	(1.2)	(1.4)	(1.2)	(1.2)	(1.2)	(1.2)	(1.2)	(1.3)	(1.3)	(1.4)	(1.0)
Three-factor alpha	−0.13	−0.07	−0.11*	−0.09	−0.09	−0.09*	−0.08	−0.03	−0.03	0.03	0.16
	(−1.3)	(−1.0)	(−1.8)	(−1.6)	(−1.6)	(−1.7)	(−1.5)	(−0.5)	(−0.5)	(0.3)	(1.0)
Four-factor alpha	−0.10	−0.05	−0.09	−0.08	−0.08	−0.08	−0.09	−0.04	−0.06	−0.02	0.08
	(−1.0)	(−0.8)	(−1.6)	(−1.5)	(−1.4)	(−1.6)	(−1.5)	(−0.6)	(−0.7)	(−0.2)	(0.5)
Five-factor alpha	−0.14	−0.12**	−0.15***	−0.15***	−0.15***	−0.15***	−0.14**	−0.07	−0.06	0.09	0.23
	(−1.6)	(−2.1)	(−2.8)	(−3.0)	(−2.8)	(−3.4)	(−2.6)	(−1.0)	(−0.7)	(0.7)	(1.3)
Noise beta (ex ante)	−0.84***	−0.40***	−0.23***	−0.11***	0.00	0.10***	0.21***	0.34***	0.52***	1.01***	1.85***
	(−11.9)	(−11.0)	(−9.3)	(−5.9)	(−0.0)	(7.7)	(16.1)	(23.7)	(28.6)	(26.0)	(19.2)
Noise beta (ex post)	0.00	0.04	0.07**	0.08*	0.09*	0.11**	0.19***	0.17***	0.21***	0.17***	0.17***
	(−0.0)	(0.9)	(2.0)	(1.8)	(1.7)	(2.0)	(3.4)	(2.6)	(3.0)	(3.0)	(2.6)
Market beta (ex post)	0.95***	0.94***	0.94***	0.95***	0.96***	0.98***	0.99***	1.00***	1.01***	1.07***	0.12**
	(20.9)	(47.0)	(60.2)	(72.5)	(78.8)	(86.1)	(93.3)	(93.6)	(80.5)	(55.8)	(2.2)
HML beta (ex post)	0.11	0.14***	0.13***	0.14***	0.13***	0.10***	0.10**	0.05	0.02	−0.13*	−0.24**
	(1.4)	(3.0)	(2.9)	(3.2)	(3.8)	(3.4)	(2.5)	(1.6)	(0.6)	(−1.9)	(−2.1)
SMB beta (ex post)	0.22***	0.14***	0.12***	0.10***	0.09**	0.11***	0.12***	0.14***	0.20***	0.38***	0.16**
	(4.0)	(3.4)	(3.8)	(3.4)	(2.0)	(3.7)	(3.6)	(3.7)	(8.0)	(10.5)	(2.1)
Age (years)	14.20	15.43	16.40	16.37	16.07	16.45	16.49	16.56	15.55	14.65	
Assets (Bn. $)	0.86	1.31	1.55	1.80	1.84	2.04	1.96	1.97	1.69	1.41	
Expense ratio (%)	1.27	1.17	1.15	1.12	1.12	1.14	1.15	1.18	1.21	1.28	
Turnover ratio (%)	94.61	79.36	76.07	75.85	75.12	74.38	76.45	79.14	84.60	98.34	

***$p < 0.01$; **$p < 0.05$; *$p < 0.1$

Table A2: Mutual fund portfolio sort on FX noise betas

This table shows excess returns, alphas, and characteristics of mutual fund portfolios formed on FX noise betas. Returns and alphas are given in monthly percent, Newey and West (1987) t-statistics are shown in parentheses. Alphas are intercepts of regression of monthly returns on risk factors. Three-factor alphas are based on the Fama and French (1993) risk factors, four-factor alphas include the additional momentum factor of Carhart (1997). The risk factors of Fama and French (2015) give rise to the five-factor alpha. Ex-ante betas are calculated as the mean of the rolling betas used to form the portfolios, ex-post betas are determined in a regression over the full time-series of portfolio returns. The last part of the table shows the average age, assets under management, expense ratio, and turnover ratio of the funds in each portfolio.

Portfolio	1	2	3	4	5	6	7	8	9	10	10–1
Excess return	0.47	0.46	0.42	0.41	0.42	0.40	0.42	0.43	0.45	0.47	0.00
	(1.4)	(1.5)	(1.3)	(1.3)	(1.3)	(1.2)	(1.2)	(1.2)	(1.2)	(1.1)	(−0.0)
Three-factor alpha	−0.05	−0.04	−0.07	−0.08	−0.07	−0.09**	−0.08*	−0.08*	−0.08	−0.10	−0.05
	(−0.6)	(−0.5)	(−1.1)	(−1.3)	(−1.2)	(−2.2)	(−1.7)	(−1.8)	(−1.3)	(−1.0)	(−0.3)
Four-factor alpha	−0.04	−0.03	−0.06	−0.07	−0.06	−0.10**	−0.09*	−0.09*	−0.10	−0.12	−0.07
	(−0.5)	(−0.4)	(−0.8)	(−1.2)	(−1.1)	(−2.1)	(−1.7)	(−1.9)	(−1.6)	(−1.3)	(−0.6)
Five-factor alpha	−0.13	−0.11*	−0.16***	−0.15***	−0.15***	−0.15***	−0.10**	−0.08*	−0.04	0.03	0.16
	(−1.6)	(−1.8)	(−2.7)	(−2.9)	(−3.0)	(−3.7)	(−2.0)	(−1.7)	(−0.5)	(0.3)	(1.1)
Noise beta (ex ante)	−0.52***	−0.26***	−0.16***	−0.09***	−0.03***	0.03***	0.09***	0.17***	0.29***	0.60***	1.11***
	(−9.6)	(−9.6)	(−9.0)	(−7.4)	(−3.2)	(3.0)	(6.9)	(8.6)	(9.3)	(9.4)	(9.7)
Noise beta (ex post)	0.02	0.05**	0.05**	0.06***	0.06***	0.05***	0.04***	0.05***	0.05***	0.06***	0.04
	(0.9)	(2.4)	(2.6)	(3.0)	(3.5)	(4.2)	(3.9)	(4.8)	(4.7)	(3.9)	(1.3)
Market beta (ex post)	0.96***	0.93***	0.94***	0.95***	0.95***	0.97***	0.99***	1.00***	1.03***	1.09***	0.14***
	(44.8)	(57.2)	(68.7)	(74.6)	(90.7)	(111.0)	(101.2)	(111.1)	(77.2)	(46.4)	(3.0)
HML beta (ex post)	0.18***	0.17***	0.19***	0.17***	0.16***	0.11***	0.06**	0.02	−0.06**	−0.15***	−0.33***
	(3.2)	(3.2)	(3.9)	(3.4)	(3.3)	(3.5)	(2.6)	(0.8)	(−2.0)	(−2.8)	(−3.3)
SMB beta (ex post)	0.20***	0.16***	0.10*	0.10*	0.08	0.10***	0.15***	0.18***	0.25***	0.36***	0.15**
	(3.1)	(2.7)	(1.8)	(1.8)	(1.6)	(2.8)	(8.8)	(13.8)	(9.8)	(11.0)	(2.3)
Age (years)	13.96	15.58	16.14	16.41	17.15	17.05	17.02	16.58	16.26	14.65	
Assets (Bn. $)	0.92	1.34	1.59	1.76	1.88	1.84	2.05	1.96	1.85	1.63	
Expense ratio (%)	1.25	1.19	1.14	1.11	1.11	1.11	1.13	1.16	1.20	1.28	
Turnover ratio (%)	82.80	76.18	75.23	73.05	72.66	73.10	78.32	79.48	86.37	100.00	

***$p < 0.01$; **$p < 0.05$; *$p < 0.1$

B Derivation of the Volatility Surface

OptionMetrics' smoothed volatility surface is derived from a kernel regression. That is, each point on the surface is given by a weighted sum of the observed implied volatilities. The weights are defined as the product of the options' vegas and the following kernel function:

$$\Phi(i,j) = \frac{1}{\sqrt{2\pi}} \exp\left(- \left(\frac{x_{ij}^2}{0.1} + \frac{y_{ij}^2}{0.01} + \frac{z_{ij}^2}{0.002} \right) \right), \tag{5}$$

where

$$\begin{aligned} x_{ij} &= \log(T_i/T_j), \\ y_{ij} &= \Delta_i - \Delta_j, \\ z_{ij} &= \begin{cases} 1, & \text{option } i \text{ and } j \text{ are both calls or puts,} \\ 0, & \text{option } i \text{ and } j \text{ are not of the same option type.} \end{cases} \end{aligned} \tag{6}$$

Thus, option similarity is quantified along the dimensions time to maturity, moneyness (given by delta), and option type.

C Details on the mutual fund sample

To construct the sample of mutual funds, we first select all US-listed mutual funds from the Morningstar database with a focus on equity investments. More precisely, we only keep funds belonging to one of the following Morningstar categories:

- US Fund Small/Mid-Cap/Large Blend
- US Fund Small/Mid-Cap/Large Value
- US Fund Small/Mid-Cap/Large Growth
- US Fund Allocation–85%+ Equity
- US Fund Preferred Stock
- US Fund Bear Market
- US Fund Market Neutral

In addition, we exclude funds of funds and index funds as indicated by the corresponding flags of the database, and funds with the word "index" in their name, in line with Pástor, Stambaugh and Taylor (2015). Finally, we exclude all monthly fund observations where the fund's age ist below 3 years or the assets under management are less than 15 million USD.

Figures

Figure 1: Exemplary implied volatilities

This figure shows observed implied volatilities of all options written on *Teladoc, Inc.* as of January 29, 2016. The grey shaded area indicates the delta-maturity range that is covered by the smoothed volatility surface provided by OptionMetrics. The blue dots correspond to the put volatilities shown in Fig. 2.

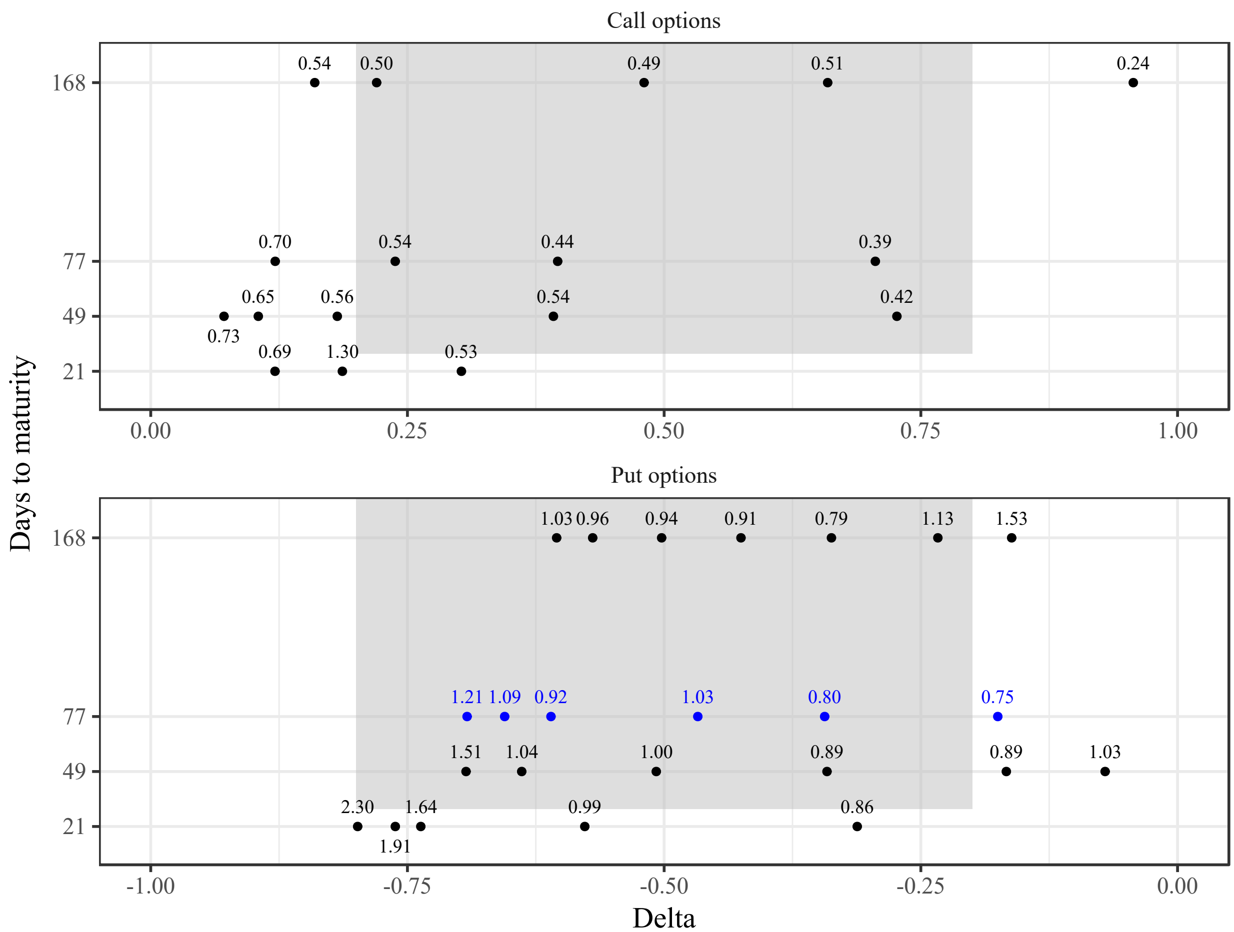


Figure 2: Exemplary construction of volatility fitting errors

This figure visualizes the construction of volatility fitting errors for a single underlying stock. The blue dots depict the implied volatilities of all put options written on *Teladoc, Inc.* with 77 days to maturity, as of January 29, 2016. The black dotted line shows the true volatility surface resulting from a kernel regression of the observed volatilities. In our dataset, this volatility surface is given on a discrete grid, as indicated by the black squares. The black solid line shows the result from a linear interpolation between these grid points, which we use to calculate the fitting errors.

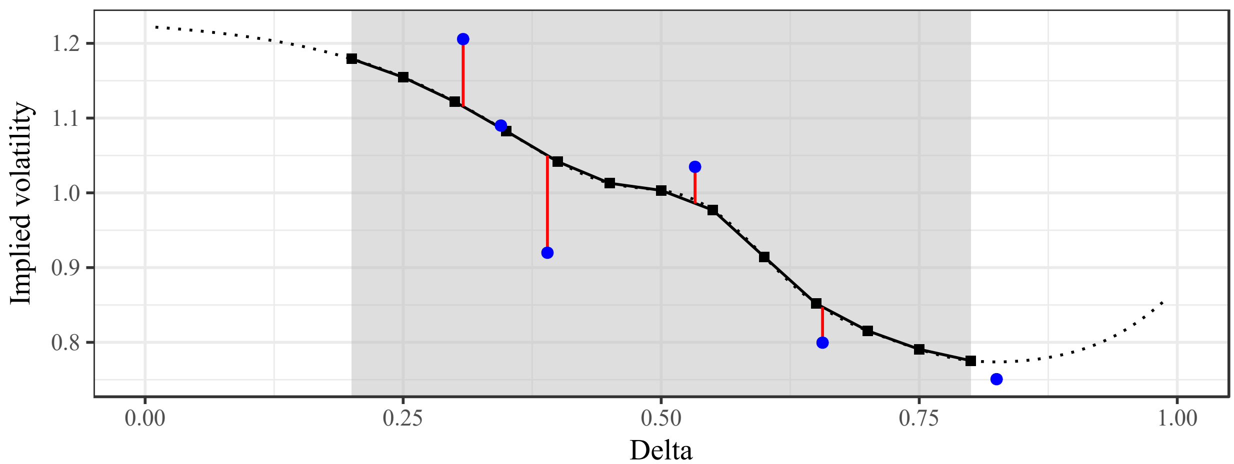


Figure 3: Examples of volatility fitting errors

This figure shows quantiles of fitting errors at different dates. On each date, we group all available options on all stocks by their respective call-equivalent delta and determine several quantiles of the deviation between the observed implied volatility and the corresponding estimate from the volatility surface. The bottom and top dashed lines show the 10% and 90% quantile, respectively. The shaded area covers the range between the 25% and 75% quantile, while the solid line indicates the median.

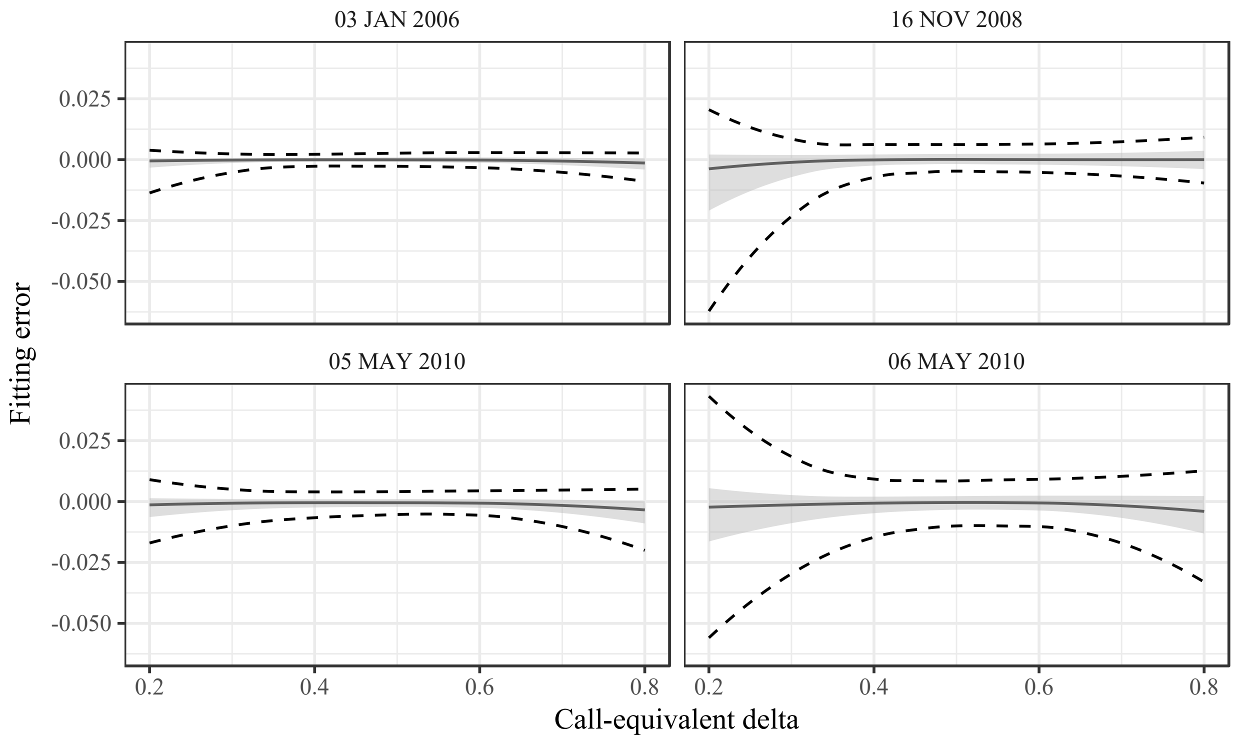
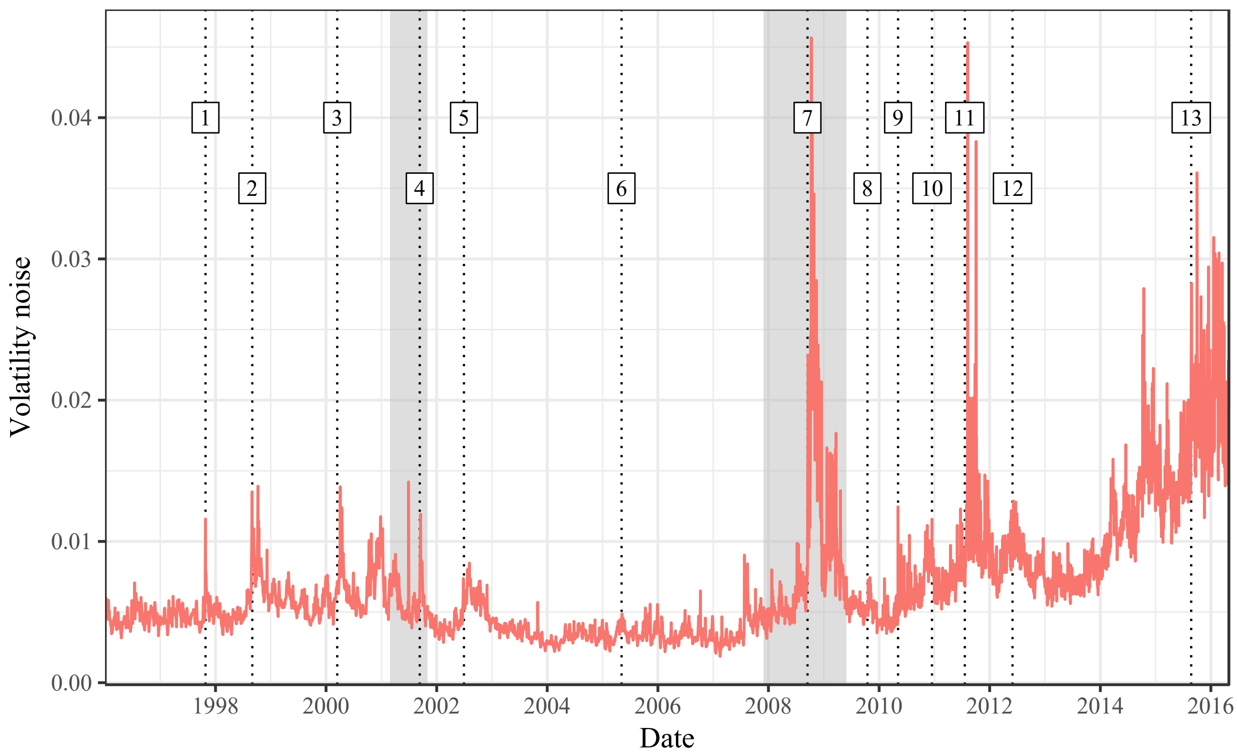


Figure 4: Time series of volatility noise

This figure visualizes volatility noise over time. The grey bars indicate NBER recessions, vertical lines show important economic events, as detailed below.

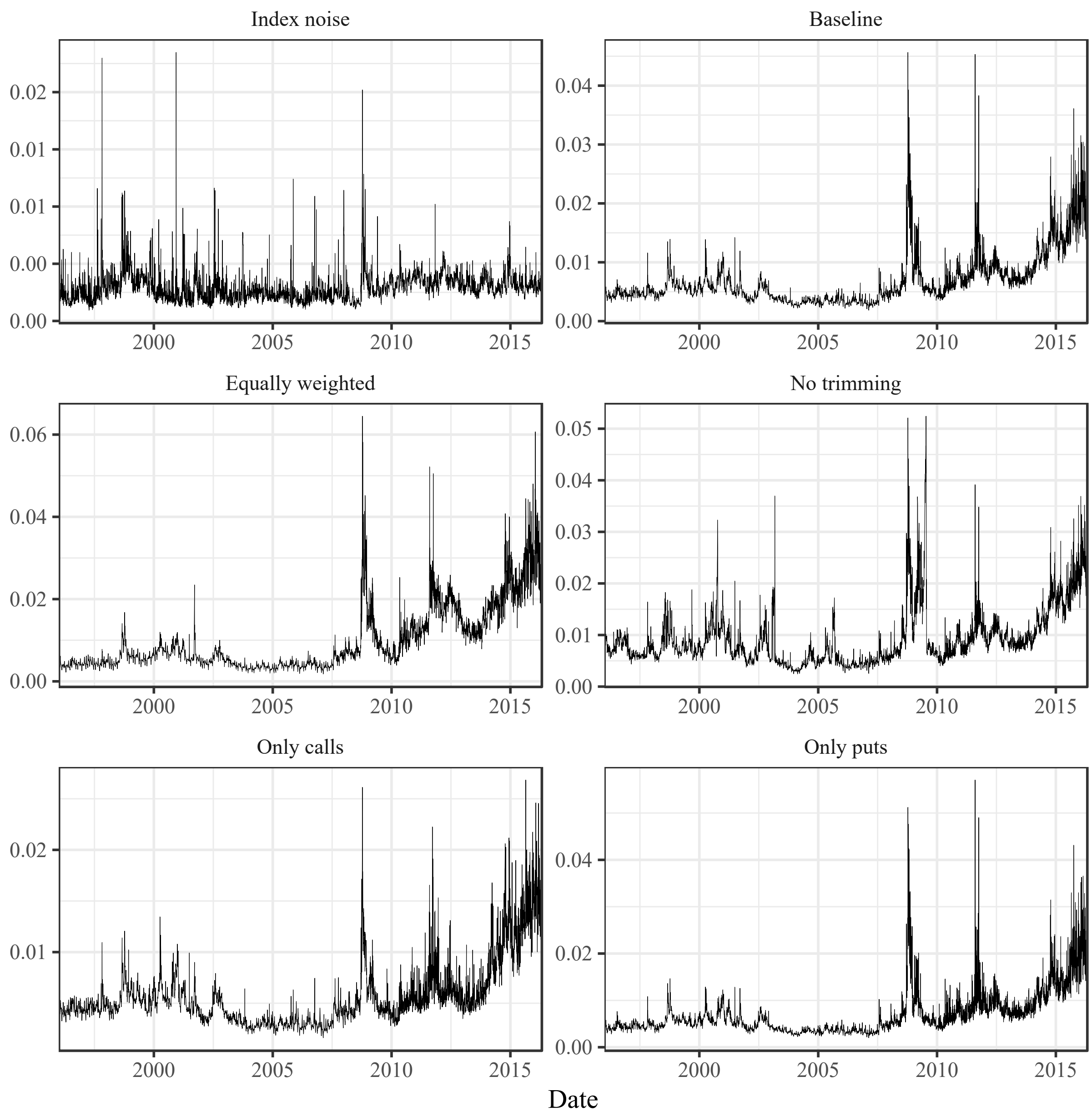


Description of events:

1. Mini-crash
2. LTCM bailout
3. Dot-com collapse
4. September 11 attacks
5. Argentine great depression
6. Downgrading of GM/Ford
7. Lehman Brothers
8. Downgrading of Greece
9. Flash crash
10. European Financial Stability Facility
11. EU debt crisis (Greece)
12. EU debt crisis (Spain/Italy)
13. Flash crash (2015)

Figure 5: Time series of alternative volatility noise measures

This figure visualizes the time-series of index noise, as shown on the top left, and several alternative specifications of (stock) volatility noise. The top right panel replicates the time-series plot of volatility noise shown in Fig. 4 for comparison. The second rows shows noise measures with equal weighting of the stock-individual fitting errors on the left, and the result from an aggregation without trimming of the 5% lowest and highest fitting errors on the right. The noise measures in the third row are solely derived from call and put options, respectively.



Tables

Table 1: Summary statistics

This table shows summary statistics on the size of the option sample after matching with the volatility surface data. We report the number of unique stocks, stock-days, and options per year, as well as the number of options per stock-day, where we only consider options with a time to maturity of less than one year and a call-equivalent delta between 0.2 and 0.8. For each of these quantities, we report the mean, standard deviation, and the 5%, 50% (median), and 95% quantile.

	Mean	Std. dev.	5%	Median	95%
Stocks per year	3 260	723	2 382	3 110	4 343
Stock-days per year	688 620	182 363	460 413	646 852	1 009 398
Options per year	16 320 387	9 061 073	7 971 290	13 337 277	33 917 132
Options per stock-day	24	30	6	16	70

Table 2: Option characteristics across moneyness and maturity

This table shows equally-weighted averages of fitting errors (in basis points), open interest, and daily trading volume as well as the number of options per moneyness-maturity category. Options are classified as out-of-the-money, at-the-money, or in-the-money if the absolute value of the options' delta lies between 0.2 and 0.4, 0.4 and 0.6, or 0.6 and 0.8, respectively.

Panel A: Average fitting error (annual volatility in bps)

	Call options			Put options		
Months to maturity	OTM	ATM	ITM	OTM	ATM	ITM
1-3	229.96	77.51	115.03	107.11	81.11	250.90
4-6	103.64	44.51	62.79	57.41	50.16	129.56
7-9	86.11	48.03	63.72	58.03	51.13	104.36
10-12	69.89	38.36	51.71	49.88	53.93	125.54

Panel B: Average open interest

	Call options			Put options		
Months to maturity	OTM	ATM	ITM	OTM	ATM	ITM
1-3	1 187.89	1 135.82	753.40	1 040.17	796.68	471.41
4-6	890.83	813.26	501.00	728.94	530.80	307.42
7-9	764.62	629.09	424.36	599.58	400.40	253.38
10-12	2 754.76	2 739.52	1 826.30	2 363.77	1 694.73	959.48

Panel C: Average trading volume

	Call options			Put options		
Months to maturity	OTM	ATM	ITM	OTM	ATM	ITM
1-3	100.26	111.99	40.46	90.08	73.09	19.73
4-6	34.65	37.92	13.31	27.25	18.78	5.01
7-9	22.87	22.78	8.75	17.79	11.45	3.29
10-12	55.97	64.57	26.69	48.96	29.99	7.46

Panel D: Number of options (millions)

	Call options			Put options		
Months to maturity	OTM	ATM	ITM	OTM	ATM	ITM
1-3	23.70	20.46	24.12	24.98	21.01	24.40
4-6	20.03	19.57	22.28	23.04	20.20	20.82
7-9	9.91	11.22	12.52	12.90	11.62	10.32
10-12	1.57	1.48	1.69	1.77	1.52	1.62

Table 3: Fitting errors and the cross-section of stocks

This table shows characteristics of stock portfolios formed on monthly averages of volatility fitting errors. The first line contains the average volatility fitting error per portfolio. In the second part of the table, we analyze several firm characterisics: market capitalization (in billion USD), the average dollar trading volume over the previous month (in million USD), the relative bid-ask spread (in percent), the illiquidity measure of Amihud (2002) (i.e., the one-month average fraction of absolute returns per dollar trading volume in 100 million USD), and idiosyncratic volatility (see Ang et al., 2006). The bottom part shows the average open interest and trading volume (both given in thousands of options), as well as the relative bid-ask spread of the associated options. To calculate these option variables, we first aggregate all for a given stock-day and form then monthly averages. The last column shows the difference between the highest and lowest portfolio with statistical significances derived from Newey and West (1987) standard errors.

Portfolio	1	2	3	4	5	6	7	8	9	10	10–1
Avg. fitting error (%)	0.21	0.32	0.41	0.49	0.59	0.71	0.87	1.12	1.59	3.83	3.62***
Market cap (bn $)	20.90	20.56	17.85	17.01	15.37	14.18	11.65	8.89	7.48	4.62	−16.28***
Stock dollar trading volume (mn $)	94.31	83.17	74.48	70.40	58.86	50.42	40.91	34.27	24.98	15.04	−79.27***
Stock bid-ask spread (%)	0.47	0.48	0.50	0.51	0.51	0.56	0.57	0.59	0.64	0.77	0.30***
Amihud illiquidity (per $100 mn)	1.20	1.14	1.30	1.51	1.73	1.95	2.29	2.66	3.51	5.77	4.57***
Idiosyncratic volatility (%)	1.28	1.48	1.62	1.77	1.91	2.07	2.23	2.41	2.61	2.85	1.57***
Open interest (thousands)	22.32	22.46	22.14	21.68	19.17	16.95	14.22	12.53	10.57	8.86	−13.45***
Option trading volume (thousands)	1.54	1.26	1.16	1.14	1.02	0.90	0.76	0.68	0.57	0.46	−1.09***
Option bid-ask spread (%)	18.55	16.01	16.31	17.14	18.39	20.18	22.35	25.33	30.49	42.94	24.39***

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

Table 4: Summary statistics on volatility noise

This table shows the mean, standard deviation, the 5%, 50% (median) and 95% quantile, and the first-order autocorrelation of the time series of volatility noise for different subsamples. The mean, standard deviation, and quantiles are given in percent.

Sample	Mean	Std	5%	Median	95%	AC
Overall	0.683	0.441	0.292	0.544	1.619	0.913
1996-1999	0.524	0.126	0.394	0.494	0.748	0.909
2000-2003	0.526	0.176	0.313	0.491	0.855	0.911
2004-2007	0.347	0.075	0.259	0.330	0.483	0.808
2008-2011	0.796	0.472	0.420	0.648	1.666	0.793
2012-2016	1.181	0.498	0.648	1.023	2.136	0.874
Expansions	0.669	0.426	0.289	0.534	1.577	0.923
Recessions	0.791	0.529	0.425	0.605	2.020	0.857

Table 5: Pairwise correlations between the noise measures and economic state variables

This table shows pairwise correlations (in percent) between changes in volatility noise, treasury noise, FX noise, and other economic state variables. The TED spread is the spread between the 3-month LIBOR rate and the T-Bill rate, which is the yield on 3-month US treasuries. Repo rate is the average rate on a 3-month general collateral repurchasement agreement. OtR premium is the on-the-run premium for 10-year bonds. Market return is the CRSP value-weighted market return, VIX is the CBOE Volatility Index. Finally, we measure aggregate market liquidity using value-weighted averages of the stock-individual relative bid-ask spreads and the Amihud (2002) measure.

Panel A: Monthly changes

	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1) Volatility noise	33	23	14	−06	−08	06	−29	40	35	33
(2) Treasury noise		21	18	−25	−25	09	−33	36	05	29
(3) FX noise			59	−21	−04	00	−17	36	18	25
(4) TED spread				−46	04	−02	−10	20	17	21
(5) T-Bill rate					72	19	18	−13	−01	−13
(6) Repo rate						07	08	−10	03	−06
(7) OtR premium							26	−18	02	−04
(8) Market return								−73	−06	−41
(9) VIX									12	39
(10) Avg. bid-ask spread										06
(11) Amihud illiquidity										

Panel B: Weekly changes

	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1) Volatility noise	02	23	10	−06	−02	−11	−22	25	08	01
(2) Treasury noise		07	15	−15	−14	03	−13	12	03	−04
(3) FX noise			15	−03	−01	−02	−07	10	17	−04
(4) TED spread				−71	03	−00	−07	13	05	−01
(5) T-Bill rate					35	16	10	−11	01	−04
(6) Repo rate						05	05	−04	06	01
(7) OtR premium							25	−22	00	−03
(8) Market return								−78	−21	−14
(9) VIX									18	08
(10) Avg. bid-ask spread										05
(11) Amihud illiquidity										

Table 6: Granger causality tests between the noise measures

This table shows χ^2-statistics and p-values of pairwise Granger causality tests. The underlying VAR models incorporate the logarithm of volatility noise (VN), treasury noise (TN) and FX noise (FXN). Each entry corresponds to a separate test with the null hypothesis that the row variable does not Granger-cause the column variable. Panel A shows result for the monthly frequency, Panel B corresponds to the weekly frequency.

Panel A: Monthly frequency

	VN	TN	FXN
VN		9.70**	9.60**
		(0.046)	(0.048)
TN	3.54		4.22
	(0.472)		(0.378)
FXN	17.77***	6.89	
	(0.001)	(0.142)	

Panel B: Weekly frequency

	VN	TN	FXN
VN		47.71***	26.98**
		(0.000)	(0.029)
TN	10.58		20.26
	(0.782)		(0.162)
FXN	39.01***	12.67	
	(0.001)	(0.628)	

***$p < 0.01$; **$p < 0.05$; *$p < 0.1$

Table 7: Mutual fund portfolio sort on volatility noise betas

This table shows excess returns, alphas, and characteristics of portfolios formed on volatility noise betas. Returns and alphas are given in monthly percent, Newey and West (1987) t-statistics are shown in parentheses. Alphas are intercepts of regression of monthly returns on risk factors. Three-factor alphas are based on the Fama and French (1993) risk factors, four-factor alphas include the additional momentum factor of Carhart (1997). The risk factors of Fama and French (2015) give rise to the five-factor alpha. Ex-ante betas are calculated as the mean of the rolling betas used to form the portfolios, ex-post betas are determined in a regression over the full time-series of portfolio returns The last part of the table shows the average age, assets under management, expense ratio, and turnover ratio of the funds in each portfolio.

Portfolio	1	2	3	4	5	6	7	8	9	10	10–1
Excess return	0.29	0.38	0.35	0.42	0.42	0.43	0.44	0.49	0.53	0.59	0.30**
	(0.8)	(1.1)	(1.1)	(1.3)	(1.3)	(1.3)	(1.3)	(1.4)	(1.5)	(1.5)	(2.1)
Three-factor alpha	−0.22***	−0.12*	−0.14**	−0.07	−0.07	−0.06	−0.05	−0.01	0.01	0.03	0.25*
	(−2.6)	(−1.7)	(−2.3)	(−1.5)	(−1.6)	(−1.1)	(−1.0)	(−0.2)	(0.1)	(0.3)	(1.8)
Four-factor alpha	−0.23***	−0.12*	−0.14**	−0.07	−0.08	−0.06	−0.05	−0.01	0.01	0.02	0.25*
	(−2.8)	(−1.8)	(−2.3)	(−1.4)	(−1.6)	(−1.0)	(−0.9)	(−0.2)	(0.1)	(0.2)	(1.9)
Five-factor alpha	−0.21**	−0.13*	−0.18***	−0.12***	−0.12***	−0.11**	−0.11**	−0.07	−0.04	0.07	0.28*
	(−2.4)	(−1.8)	(−3.0)	(−2.6)	(−2.9)	(−2.2)	(−2.1)	(−1.3)	(−0.5)	(0.6)	(1.8)
Noise beta (ex ante)	−3.72***	−1.77***	−1.00***	−0.44***	0.02	0.45***	0.89***	1.41***	2.13***	4.06***	7.78***
	(−11.7)	(−10.6)	(−9.1)	(−5.8)	(0.4)	(6.6)	(9.9)	(11.8)	(12.9)	(13.8)	(13.7)
Noise beta (ex post)	0.28	0.28	0.42	0.49**	0.36*	0.51*	0.45**	0.65*	0.69*	0.98**	0.70*
	(0.6)	(1.0)	(1.4)	(2.2)	(1.8)	(1.7)	(2.0)	(1.8)	(1.6)	(2.2)	(1.9)
Market beta (ex post)	0.97***	0.96***	0.97***	0.96***	0.97***	0.97***	0.98***	0.98***	0.99***	1.04***	0.07*
	(40.0)	(63.3)	(60.6)	(81.3)	(103.0)	(104.2)	(85.2)	(89.4)	(65.2)	(66.8)	(1.7)
HML beta (ex post)	−0.02	0.04	0.08**	0.09***	0.10***	0.12***	0.12***	0.12***	0.12**	0.03	0.05
	(−0.4)	(0.9)	(2.0)	(3.1)	(4.0)	(3.6)	(3.1)	(3.0)	(2.5)	(0.5)	(0.6)
SMB beta (ex post)	0.30***	0.22***	0.14***	0.12***	0.10***	0.10***	0.10***	0.10***	0.15***	0.28***	−0.02
	(10.8)	(12.0)	(4.2)	(3.6)	(3.6)	(3.0)	(2.8)	(2.7)	(4.5)	(9.6)	(−0.5)
Age (years)	14.71	16.36	16.74	16.82	16.88	16.86	16.30	15.96	15.60	14.25	
Assets (Bn. $)	1.03	1.55	1.81	1.88	2.03	2.05	2.00	1.74	1.52	1.18	
Expense ratio (%)	1.27	1.19	1.15	1.13	1.11	1.11	1.12	1.14	1.18	1.28	
Turnover ratio (%)	99.84	83.81	78.76	75.78	73.63	72.64	72.96	76.41	77.88	87.27	

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

Table 8: Fama-MacBeth regressions of mutual fund returns

This table shows results from Fama-MacBeth regressions of monthly mutual fund excess returns on ex-post portfolio betas and control variables. We control for the funds' age (in months), size (given by the logarithm of the assets under management), the expense ration and turnover ratio. The first two regressions correspond to volatility noise, models (3) and (4) to treasury noise, and the last two columns show the results for FX noise. We report time-series averages of the cross-sectional regression coefficients in percent, along with Newey and West (1987) t-statistics.

	Excess return					
	Volatility noise		Treasury noise		FX noise	
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	−0.88	0.10	−0.06	−0.07	0.81	1.41
	(−0.7)	(0.1)	(−0.1)	(−0.1)	(1.0)	(1.6)
Noise beta	0.19**	0.18***	0.06	−0.01	−0.04	0.07
	(2.4)	(2.7)	(0.2)	(−0.0)	(−0.0)	(0.1)
Market beta	1.10	0.80	0.54	1.19	−0.47	−0.46
	(0.9)	(0.7)	(0.4)	(1.1)	(−0.5)	(−0.5)
SMB beta	0.33	0.35	0.18	0.35	0.56	0.54*
	(0.6)	(0.8)	(0.3)	(0.6)	(1.6)	(1.8)
HML beta	1.16	0.97	−0.31	0.16	0.06	0.02
	(1.1)	(1.0)	(−0.3)	(0.2)	(0.1)	(0.0)
Size		−0.03*		−0.03*		−0.02**
		(−1.7)		(−2.0)		(−2.0)
Age		0.00		0.00		0.00
		(−0.2)		(−0.6)		(−0.8)
Expense ratio		−0.09*		−0.08		−0.10**
		(−1.9)		(−1.5)		(−2.1)
Turnover ratio		0.00		0.00		0.00
		(−0.2)		(−0.2)		(−0.3)
Adj. R^2	0.04	0.08	0.05	0.09	0.06	0.09

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

Table 9: Summary statistics on alternative specifications of volatility noise

This table shows the mean, standard deviation, the 5%, 50% (median) and 95% quantile, and the first-order autocorrelation corresponding to alternative volatility noise specifications. In addition, the last two columns show the correlation between monthly changes of the respective volatility noise measure and monthly changes in treasury noise and FX noise, respectively. The first line corresponds to the baseline specification of volatility noise, the second line shows results for volatility noise constructed from index options. In the remaining lines, we change one aspect of the baseline specification at a time: the weighting scheme, trimming of extreme fitting errors, and the subset of options (i.e., calls or puts) over which we form the root-mean-square error. The mean, standard deviation, and quantiles are given in percent.

Specification	Mean	Std	5%	Median	95%	AC	VN corr.	TN corr.	FXN corr.
Baseline	0.68	0.44	0.29	0.54	1.62	0.91	1.00	0.31	0.23
Index noise	0.29	0.13	0.15	0.27	0.48	0.59	0.13	0.00	0.04
Equally weighted	0.98	0.79	0.31	0.64	2.65	0.95	0.88	0.25	0.28
No trimming	0.92	0.55	0.38	0.76	2.03	0.93	0.51	0.11	0.20
Only calls	0.57	0.32	0.26	0.48	1.30	0.91	0.57	0.19	0.14
Only puts	0.70	0.48	0.30	0.55	1.64	0.84	0.93	0.29	0.22

Table 10: Pairwise correlations between index noise and economic state variables

This table shows pairwise correlations (in percent) between monthly changes in index noise, the baseline (stock) volatility noise, treasury noise and changes in other economic state variables as well as stock market returns. The TED spread is the spread between the 3-month LIBOR rate and the T-Bill rate, which is the yield on 3-month US treasuries. Repo rate is the average rate on a 3-month general collateral repurchasement agreement. OtR premium is the on-the-run premium for 10-year bonds. Market return is the CRSP value-weighted market return, VIX is the CBOE Volatility Index. Finally, we measure aggregate market liquidity using value-weighted averages of the stock-individual relative bid-ask spreads and the Amihud (2002) measure.

	Index noise	Stock noise
Stock noise	21	
Treasury noise	05	33
FX noise	05	23
TED spread	−02	14
T-Bill rate	05	−06
Repo rate	06	−08
OtR premium	−01	06
Market return	−14	−29
VIX	18	40
Average bid-ask spread	16	35
Amihud illiquidity	−08	33

Table 11: Long-short portfolios based on alternative volatility noise specifications

This table shows monthly average excess returns and alphas from equity mutual fund long-short portfolios formed on betas corresponding to different volatility noise specifications. The first two columns show results for volatility noise derived from S&P 500 index options and the baseline specification based on stock options, respectively. In the third specification, stock-individual fitting errors are equally weighted instead of value weighting. In the fourth column, we show the results for a noise measure without the removal of the 5% lowest and highest fitting errors. Specifications five and six result from the construction of volatility noise solely from call and put options, respectively. Returns and alphas are given in monthly percent, Newey and West (1987) t-statistics are shown in parentheses. Alphas are intercepts of regression of the monthly excess returns on risk factors. Three-factor alphas are based on the Fama and French (1993) risk factors, four-factor alphas include the additional momentum factor of Carhart (1997). The risk factors of Fama and French (2015) give rise to the five-factor alpha.

		Stock noise				
	Index noise	Baseline	Equ. weighted	No trimming	Only calls	Only puts
Excess return	−0.02	0.30**	0.41**	0.04	0.30*	0.33*
	(−0.1)	(2.1)	(2.5)	(0.4)	(1.8)	(1.8)
Three-factor alpha	0.02	0.25*	0.35**	0.04	0.25	0.29*
	(0.2)	(1.8)	(2.2)	(0.4)	(1.6)	(1.7)
Four-factor alpha	0.07	0.25*	0.35**	0.06	0.23	0.27*
	(0.8)	(1.9)	(2.4)	(0.6)	(1.6)	(1.7)
Five-factor alpha	−0.03	0.28*	0.35**	−0.04	0.32*	0.38**
	(−0.3)	(1.8)	(2.0)	(−0.4)	(1.8)	(2.0)

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$